Appendix

Semi-Pooling Equilibria

**Lemma 2:** When \( R_l < 0 \) and \( Pr(h|\neg f) = 0 \), \( S_I(\neg f) = \neg v \).

*Proof.* Given that \( Z > 0 \) and \( D > 0 \), the expected utility of \( S_I(v|l) < 0 \), while the expected utility of \( S_I(\neg v|l) = 0 \). If \( Pr(h|\neg f) = 0 \), then \( Pr(l|\neg f) = 1 \). Thus, \( S_I(v|\neg f) < 0 \), and \( S_I(v|\neg f) < S_I(\neg v|\neg f) \).

**Proposition 9:** When \( R_l \leq 0 \), \( G_l > Y \) and \( p \leq \frac{R_h}{R_l-R_h} \), the strategy profiles \( S_C(h) = f \), \( S_C(l) = (f \text{ with probability } q) \), \( S_I(f) = (v \text{ with probability } x) \), and \( S_I(\neg f) = \neg v \) are in equilibrium.

*Proof.* Assume that Lemma 1 is true (see proof below). \( R_l \leq 0 \) must hold true for \( I \) to play \( \neg v \) with positive probability. \( I \) plays a mixed strategy following \( f \), and in order for that strategy to be played in equilibrium, \( I \) must be indifferent between \( v \) and \( \neg v \), \( EU_{h|f} + EU_{l|f} = 0 \). Let \( q \) represent the probability that \( f \) is played by \( C \) in state \( l \). Using Bayes’ Theorem, we derive the following probabilities: \( Pr(h|f) = \frac{p}{p+q(1-p)} \) and \( Pr(l|f) = \frac{q(1-p)}{p+q(1-p)} \). Therefore, in order for \( I \) to be indifferent between \( v \) and \( \neg v \) when \( f \) is played, \( \frac{ZR_h p}{p+q(1-p)} + \frac{ZR_q (1-p)}{p+q(1-p)} = 0 \). Solving for \( q \), we find that \( I \) is indifferent when \( q^* = \frac{-R_h p}{R_l(1-p)} \). Given Lemma 2, \( R_l \leq 0 \) and \( S_C(h) = f \), \( I \) always plays \( \neg v \) in response to \( \neg f \). \( C \) is indifferent between strategy \( f \) and \( \neg f \) in state \( l \), when the expected utility of instituting financial reforms equals the expected utility of not instituting financial reforms. Let \( x \) represent the probability that \( I \) plays \( v \) in response to \( f \). \( C \) receives a payoff of 0 when playing \( \neg f \). Let \( x \) represent the probability that \( I \) plays \( v \) in response to \( f \). In order for \( C \) to be indifferent between \( f \) and \( \neg f \), the following equality must hold: \( xG_l - Y = 0 \). Solving for \( x \), we find the following indifference condition: \( x^* = \frac{Y}{G_l} \).

By definition, \( Y > 0 \), thus, \( x^* > 0 \). In order for \( x^* \) to hold in equilibrium, \( x < 1 \), which is true when \( G_l > Y \). In order for \( C \) to prefer strategy \( f \) in equilibrium when in state \( h \),

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1 \quad \text{Cases where only when } R_l = 0 \text{ would an equilibrium exist are excluded from the set of semi-pooling equilibria that are analyzed.}
\]
Proof. \( I \) always plays \( v \) in response to \( f \) since \( R_h > 0 \), and \( \Pr[h|f] = 1 \). \( I \) plays a mixed strategy following \( \neg f \), and in order for that strategy to be played in equilibrium, \( I \) must be indifferent between \( v \) and \( \neg v \), \( EU_h|\neg f + EU_{|\neg f} = 0 \). Using Bayes’ Theorem, we derive the following probabilities: \( \Pr(h|\neg f) = \frac{pq}{pq+(1-p)} \) and \( \Pr(l|\neg f) = \frac{1-p}{pq+(1-p)} \). Therefore, in order for \( I \) to be indifferent between \( v \) and \( \neg v \) when \( \neg f \) is played, \( \frac{ZDR_hpq}{pq+(1-p)} + \frac{ZDR_l(1-p)}{pq+(1-p)} = 0 \). Solving for \( q \), we find that \( I \) in indifferent when \( q^* = \frac{-R_l(1-p)}{R_hp} \). Given that \( q^* \leq 1 \), \( \frac{-R_l(1-p)}{R_hp} \leq 1 \). This only holds true when \( p \geq \frac{R_l}{R_l-R_h} \). In order for \( C \) to prefer equilibrium play when in state \( h \), \( C \) must be indifferent between \( f \) and \( \neg f \). \( C \) receives a payoff of \( G_h - Y \) when playing \( f \), and a payoff of \( xDG_h + (1-x)(0) \) when playing \( \neg f \). Thus, \( xDG_h + (1-x)(0) = G_h - Y \) in equilibrium. Solving for \( x \), we find the following indifference curve: \( x^* = \frac{G_h-Y}{DG_h} \). Since \( x \leq 1 \), this holds true when \( \frac{G_h-Y}{DG_h} \leq 1 \). Solving for \( Y \), we find that the following condition must hold: \( Y \geq (1-D)G_h \). \( C \) must also prefer equilibrium play when in state \( L \). In order for this to be true, \( EU_{\neg f|l} \geq EU_{f|l} \). Thus, \( xDG_l > G_l - Y \). Solving for \( x \), \( x = \frac{G_h-Y}{DG_h} \), and substituting for \( x^* \), we find that the following must hold true \( \frac{G_h-Y}{DG_h} \geq \frac{G_h-Y}{DG_h} \). Since \( G_h > G_l > 0 \) and \( D > 0 \), this holds for all values of \( x^* \).

Proofs

Proof of Lemma 1. \( I \) receives a payoff of 0 when playing \( \neg v \) in response to either action by \( C \). Thus, in order for player \( I \) to prefer \( v \) to \( \neg v \), the expected payoff in equilibrium of playing \( v \) must be greater than 0. \( R_h \) is by definition greater than 0, and, thus, the payoff from \( v \) must be greater than 0. Therefore, when \( I \) assigns a probability of 1 to the state of
the world being \(h\), she will always play \(v\) in equilibrium. Given that \(R_h > R_l\), when \(R_l > 0\), \(I\) will also always play \(v\) in response to any action by \(C\) and for all beliefs regarding the state of the world.

**Proof of Proposition 1.** Given Lemma 1, an investor will only play \(v\) when assigning a probability of 1 to state \(l\) when \(R_l > 0\), and will always play \(v\) when the of probability of being in state \(h\) is 1. \(Pr(h|f) = 1, Pr(l|\neg f) = 1\). Thus, in order for \(I\) to play \(v\) in response to \(f\) when \(Pr(l|\neg f) = 1\), the following condition must be met: \(R_l \geq 0\). Given this condition, \(I\) will always prefer to play \(v\). Thus, whether \(C\) plays \(f\) or \(\neg f\) hinges on whether the added value financial institutional development justifies the costs associated with development. In order for this separating equilibrium to hold, \(C\) must prefer to play \(f\) when in state \(h\), and prefer \(\neg f\) when in state \(l\). In order for this to hold true, \(DG_h \leq G_h - Y\) and \(DG_l \geq G_l - Y\). Thus, the strategy set \(S_C(h) = f, S_C(l) = \neg f\) and \(S_I(f) = S_I(\neg f) = v\) is in equilibrium when \(\frac{G_l - Y}{G_l} \leq D \leq \frac{G_h - Y}{G_h}\) and \(R_l \geq 0\).

**Proof of Proposition 2.** Given Lemma 1, \(I\) will always play \(v\) when assigning a probability of 1 to the state of the world being \(h\), and will never play \(v\) when the probability of being in state \(l\) is 1 if \(R_l < 0\). \(Pr(h|f) = 1, Pr(l|\neg f) = 1\). In order for \(S_I(\neg f) = \neg v\) to be played in equilibrium, the following must hold true \(R_l \leq 0\). \(S_I(f) = v\) will always be played since \(Pr(h|f) = 1\). \(C\) will always prefer to play \(f\) when in state \(h\) given \(I\)'s strategy set since \(G_h - Y\) is always greater than 0. In order for \(C\) to prefer \(\neg f\) when in state \(l\), \(G_l - Y\) must be less than or equal to 0. Given that \(G_l \geq Y\), this only true when \(G_l = Y\). Thus, in order for \(S_C(h) = f, S_C(l) = \neg f, S_I(f) = v\) and \(S_I(\neg f) = \neg v\) to be played in equilibrium, \(R_l \leq 0\) and \(G_l \geq Y\).

**Proof of Proposition 3.** Assume that \(I\) plays \(v\) against both \(f\) and \(\neg f\). In order for \(C\) to play \(f\) in equilibrium when in state \(l\), \(G_l - Y \geq DG_l\). In order for \(C\) to play \(\neg f\) in equilibrium when in state \(h\), \(G_h - Y \leq DG_h\). This means that \(D \leq \frac{G_l - Y}{G_l}\) and \(D \geq \frac{G_h - Y}{G_h}\). This would imply that \(\frac{G_l - Y}{G_l} > \frac{G_h - Y}{G_h}\), which can never hold true since \(G_h > G_l\) by definition. Given
that \(Pr(h|\neg f) = 1\), we know that \(I\) will always play \(v\) in response to \(\neg f\), so the only other strategy that could be played by \(I\) is \(S_I(\neg f) = v\) and \(S_I(f) = \neg v\). Since \(D\), \(Y\) and \(G_l\) are always positive, \(C\) will always prefer the payoff \(DGl\) to \(\neg Y\) in equilibrium. Thus, \(C\) will never play \(f\) when in state \(l\) if \(I\) will respond with \(\neg v\). In fact, \(C\) will never play \(f\) in response to \(\neg v\), whether the state is \(h\) or \(l\).

**Proof of Proposition 4.** Assume that Lemma 1 holds true, and \(R_l \geq 0\). \(I\) can never increase her payoff by choosing \(\neg v\). When \(R_l \leq 0\), the probability that the state of the world is \(h\) must be high enough to justify \(I\) playing \(v\). Given that \(C\) plays \(f\) in both states, the utility \(I\) derives from playing \(v\) is \(p(ZR_h) + (1 - p)(ZR_l)\), and in order for it to be weakly preferred to \(\neg v\), \(p(ZR_h) + (1 - p)(ZR_l) \geq 0\). This holds true when \(p \geq \frac{-R_l}{R_h - R_l}\). \(I\) must also prefer \(v\) to \(\neg v\) in response to \(\neg f\) in order for this strategy set to be played in equilibrium. For this to be true, the following inequality must be satisfied: \((Pr[h|\neg f])ZDR_h + (1 - Pr[h|\neg f])\frac{ZDR_l}{D} \geq 0\). Thus, when \(Pr[h|\neg f] \geq \frac{-R_l}{R_h D^2 - R_l}\), \(I\) will prefer to play \(v\) in response to \(\neg f\). Given that \(I\) will play \(v\) in response to either \(f\) or \(\neg f\), \(C\) will only choose \(f\) when the growth benefits associated with financial institutions outweigh their costs. Since \(G_l < G_h\), \(\frac{G_l - Y}{G_l} < \frac{G_h - Y}{G_h}\).

In order for \(C\) to prefer \(f\) in both states of the world, given \(I\)'s strategy set, the following inequality must hold true: \(G_l - Y \geq DG_l\). This will hold true when \(D \leq \frac{G_l - Y}{G_l}\).

**Proof of Proposition 5.** Given Lemma 1, \(I\) will only choose \(\neg v\) as a strategy in equilibrium when \(R_l \leq 0\). Thus, this equilibrium can only exist when \(R_l \leq 0\). Given that \(C\) plays \(f\) in both states, the utility \(I\) derives from playing \(v\) is \(p(ZR_h) + (1 - p)(ZR_l)\), and in order for it to be weakly preferred to \(\neg v\), \(p(ZR_h) + (1 - p)(ZR_l) \geq 0\). This holds true when \(p \geq \frac{-R_l}{R_h - R_l}\). \(I\) must also prefer \(\neg v\) to \(v\) in response to \(\neg f\) in order for this strategy set to be played in equilibrium. For this to be true, the following inequality must be satisfied: \(Pr[h|\neg f](ZDR_h) + (1 - Pr[h|\neg f])\frac{ZDR_l}{D} \leq 0\). Thus, when \(Pr[h|\neg f] \leq \frac{-R_l}{R_h D^2 - R_l}\), \(I\) will prefer to play \(\neg v\) in response to \(\neg f\). Given that \(I\) will play \(v\) in response to \(f\) and \(\neg v\) in response to \(\neg f\), \(C\) will always prefer to play \(f\). This is because \(D\) and \(G\) are always positive, and
$G_l \geq Y$, and the utility derived from playing $\neg f$ is 0. Thus, $f$ will always be preferred in equilibrium given $I$’s strategy set.

**Proof of Proposition 6.** Assume that Lemma 1 holds true, and $R_l \geq 0$. $I$ can never increase her payoff by choosing $\neg v$, which always results in a payoff of 0, and, thus, will always play $v$. When $R_l \leq 0$, the probability that the state of the world is $h$ must be high enough to justify $I$ playing $v$. Given that $C$ plays $\neg f$ in both states, the expected utility $I$ derives from playing $v$ is $pZD R_h + (1 - p)\frac{ZD R_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZD R_h + (1 - p)\frac{ZD R_l}{D} \geq 0$. This holds true when $p \geq \frac{-R_l}{R_h D^2 - R_l}$. $I$’s off-the-equilibrium-path belief must also support playing $v$ for this strategy to hold. Thus, $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \geq 0$, and, therefore, the following off-the-equilibrium-path belief is required: $Pr[h|f] \geq \frac{-R_l}{R_h D^2 - R_l}$. Given that $I$ will play $v$ in response to either $f$ or $\neg f$, $C$ will only choose $\neg f$ when the growth benefits associated with financial institutions are overshadowed by their costs. Since $G_l < G_h$, $\frac{G_l - Y}{G_l} < \frac{G_h - Y}{G_h}$. In order for $C$ to prefer $\neg f$ in both states of the world, given $I$’s strategy set, the following inequality must hold true: $G_h - Y \leq DG_h$. This will hold true when $D \geq \frac{G_l - Y}{G_h}$.

**Proof of Proposition 7.** Given Lemma 1, $I$ will only choose $\neg v$ as a strategy in equilibrium when $R_l \leq 0$. Thus, this equilibrium can only exist when $R_l \leq 0$. Given that $C$ plays $\neg f$ in both states, the utility $I$ derives from playing $v$ is $pZD R_h + (1 - p)\frac{ZD R_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZD R_h + (1 - p)\frac{ZD R_l}{D} \leq 0$. This holds true when $p \leq \frac{-R_l}{R_h D^2 - R_l}$. $I$’s off-the-equilibrium-path belief must also support playing $\neg v$ for this strategy to hold. Thus, $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \leq 0$, and, therefore, the following off-the-equilibrium-path belief is required: $Pr[h|f] \leq \frac{-R_l}{R_h D^2 - R_l}$. Since $I$ always plays $v$, $C$ will always receive a payoff of $-Y$ when playing $f$, and 0 when playing $\neg f$. Since $Y > 0$, $C$ will always prefer $\neg f$ in equilibrium.

**Proof of Proposition 8.** Given Lemma 1, $I$ will only choose $\neg v$ as a strategy in equilibrium.
when $R_l \leq 0$. Thus, this equilibrium can only exist when $R_l \leq 0$. Since $C$ plays $\neg f$ in both states, the utility $I$ derives from playing $v$ is $pZRDR_h + (1 - p)\frac{ZRDR_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZRDR_h + (1 - p)\frac{ZRDR_l}{D} \geq 0$. This holds true when $p \geq \frac{-R_l}{R_hD - R_l}$. In order for this equilibrium to hold, $I$ must prefer to play $\neg v$ in response $f$. Note that weak consistency is not required for off-the-equilibrium-path beliefs. Thus, we can assign any belief that satisfies the following inequality: $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \leq 0$, and, therefore, the following off-the-equilibrium-path belief is required: $Pr[h|f] \leq \frac{-R_l}{R_h - R_l}$. Since player $C$ will receive a payoff of $-Y$ for playing $f$, and receives a positive payoff from playing $\neg f$ when in both state $h$ and $l$, $C$ will always play $\neg f$ in this equilibrium.